STiCM

Select / Special Topics in Classical Mechanics

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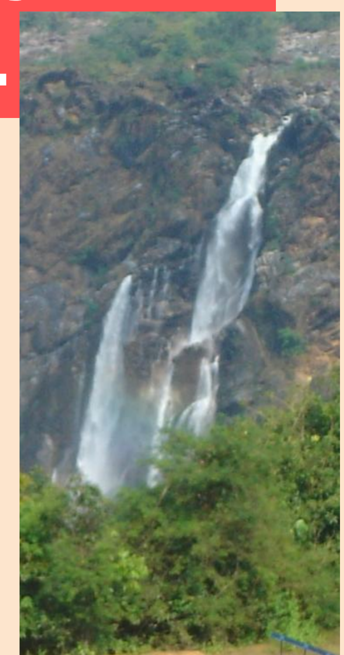
ready for take-off?

Suppose you wish to understand how a rocket lifts off against gravity…

or, track the ball hit by Sachin ..



… or, determine why water flows down the way it does..

…. one can feel quite lost !

[redacted]

[redacted]

Time

'SCIENCE' is relatively young!
Just a few hundred years old,
perhaps a few thousand years or so…..

Big Bang ~ between 12 and 14 billion years ago

The Solar System ~ 4.5 billion years old

Humans ~ few million years

Planck Era

10^{-43} seconds

Nucleo-synthesis (protons, neutrons, …..)

3 minutes

Nuclei (plasma formations; hydrogen, helium nuclei, ….)

300,000 years

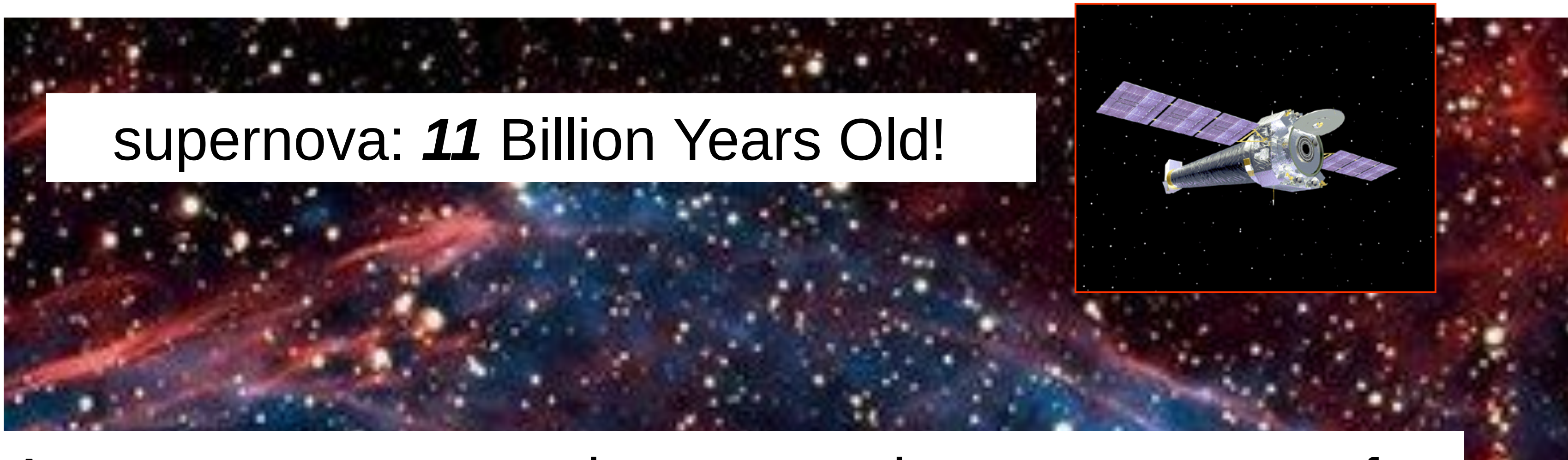
Atoms (plasma, atoms, formation of stars…..)

1 Billion years

Galaxies

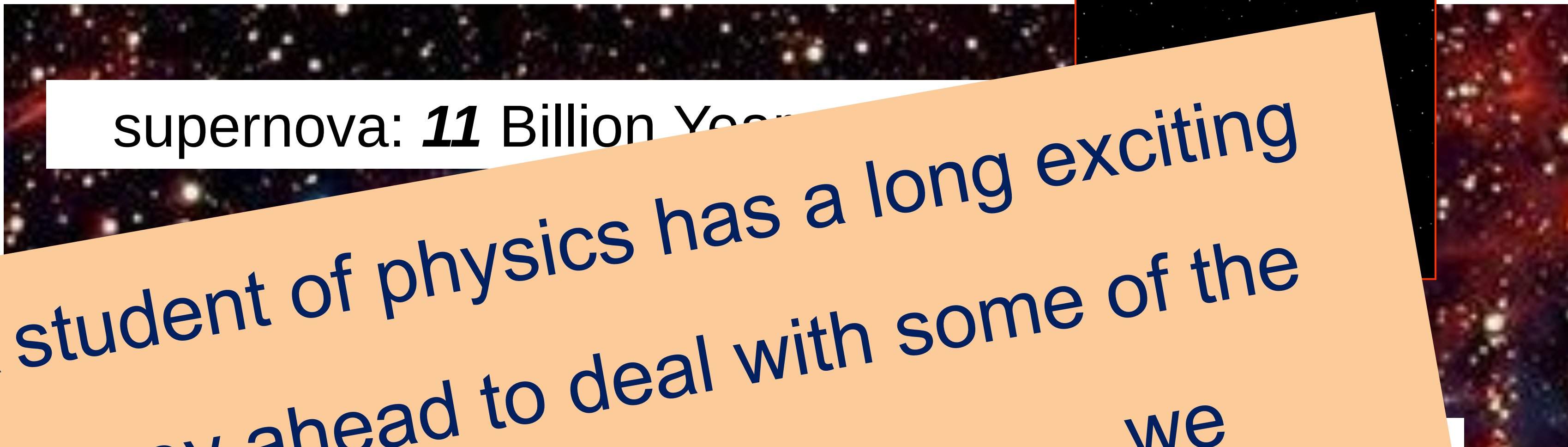
Present

supernova: ***11*** Billion Years Old!



A supernova occurs when a massive star runs out of fuel, collapses upon itself under the force of its own gravity to become a tiny, ultra-dense object called a neutron star. The star then explodes, sending out a shock wave that reverberates around the galaxy.

Where is CHANDRA?: http://science.nasa.gov/realtime/



A student of physics has a long exciting journey ahead to deal with some of the most challenging questions…… we begin this delightful voyage with the ‘beginnings’ by undertaking studies of introductory select/special topics in classical mechanics…

Select / Special Topics in Classical Mechanics

This class:

Brief overview of the course contents of the Eleven Units

Central problem in MECHANICS:

How do you characterize

the ‘state’ of a mechanical system,

and

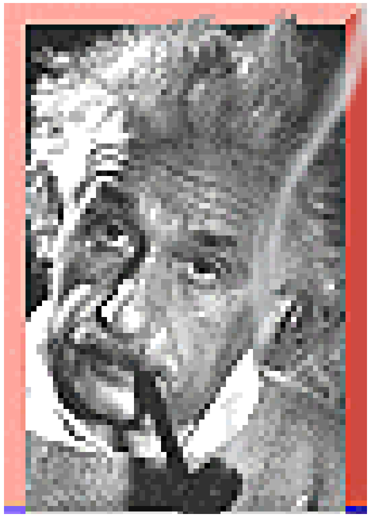
how does this state evolve with time?

CLASSICAL / QUANTUM

MECHANICS

Why do objects ‘fall’ ?

“The earth is the natural abode of things, objects ‘fall’ when thrown up just as horses return to their stables.”



"Gravity is not responsible for people falling in love"
- Albert Einstein

A table, a chair, a bowl of fruit and a violin; what else does a man need to be happy?
- Albert Einstein
1879 - 1955

$$\vec{F} = m\vec{a}$$

Unit 1: Equations of Motion.

Principle of Causality and Newton’s I & II Laws.

Determination of Newton’s 3rd Law from translational symmetry.

Application to SHO.

Linear Response Formalism
Cause-Effect Relationship

Albert Einstein: "We owe a lot to Indians, who taught us how to count, without which no worthwhile scientific discovery could have been made."

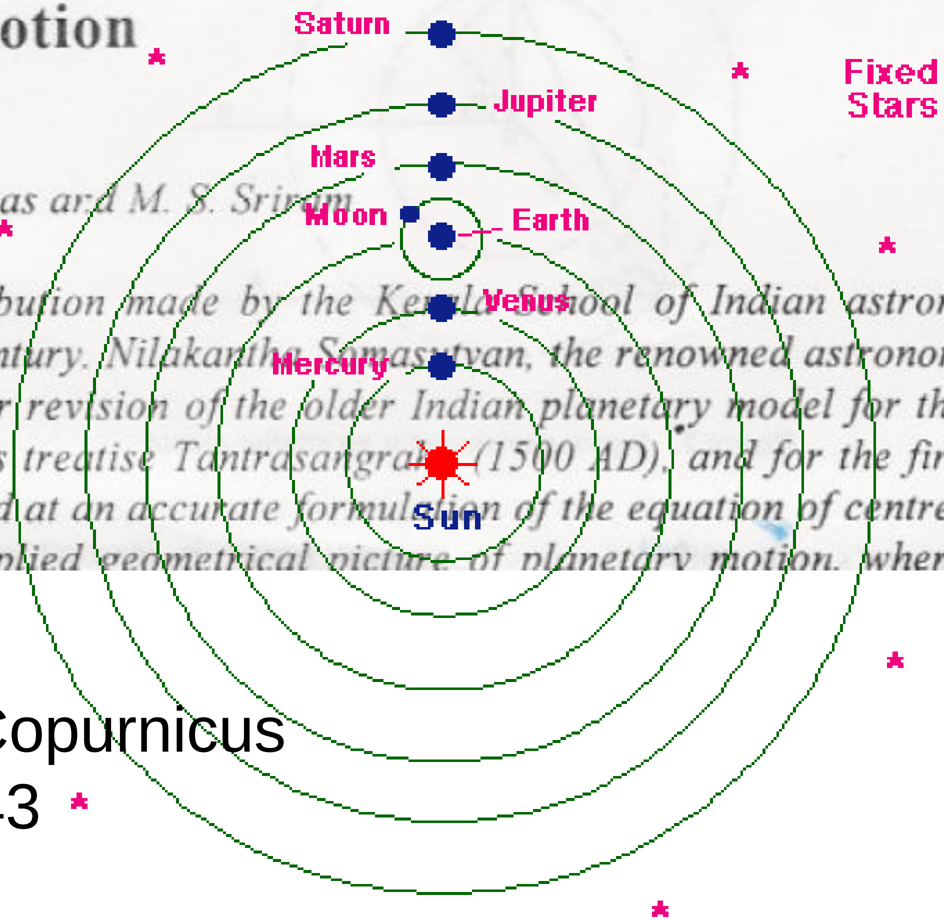
ARYABHATTA (in 5th century) introduced new concepts: sphericity of the earth, rotation about its axis, revolution around the sun, explanation of eclipses….. estimated length of the year…..

BRAHMAGUPTA (in 7th century) estimated the circumference of the earth to be around 5000 yoganas which in today's units is close to the correct value as we know it now….

Modification of the earlier Indian planetary theory by the Kerala astronomers (c. 1500 AD) and the implied heliocentric picture of planetary motion

K. Ramasubramanian, M. D. Srinivas and M. S. Sriram

We report on a significant contribution made by the Kerala School of Indian astronomers to planetary theory in the fifteenth century. Nilakantha Somasutvan, the renowned astronomer of the Kerala School, carried out a major revision of the older Indian planetary model for the interior planets, Mercury and Venus, in his treatise Tantrasangraha (1500 AD), and for the first time in the history of astronomy, he arrived at an accurate formulation of the equation of centre for these planets. He also described the implied geometrical picture of planetary motion, where the five



Nicolus Copurnicus
1473-1543

http://www.physics.iitm.ac.in/~labs/amp/kerala-astronomy.pdf

Central problem in 'Mechanics': How is the 'mechanical state' of a system described, and how does this 'state' evolve with time?

'position' and 'velocity': both needed to specify the mechanical state of a system?

The mechanical state of a system is characterized by its position and velocity, $(q,\dot{q})$

or, position and momentum, (q,p)

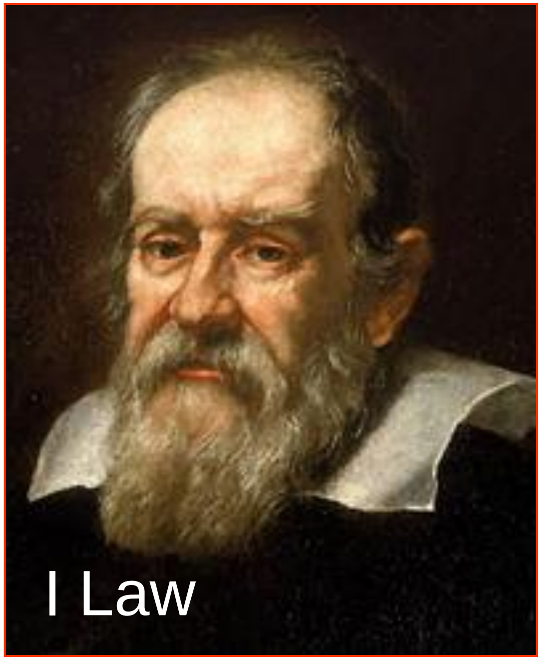
Or, equivalently by their well-defined functions:

$$L(q,\dot{q}) : Lagrangian$$
$$H(q,p) : \ Hamiltonian$$

What is 'equilibrium'?

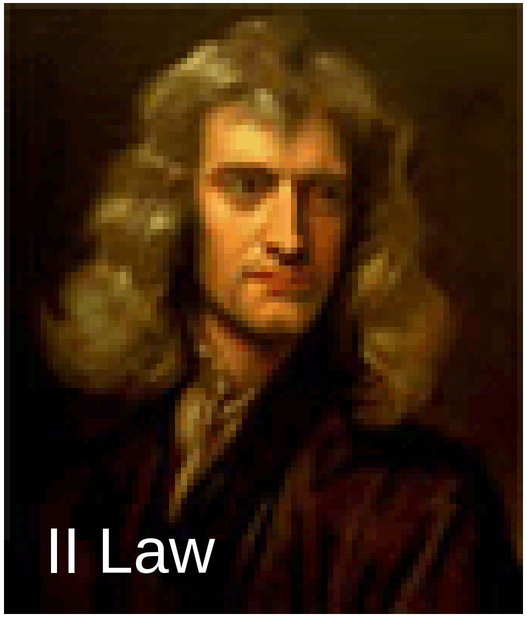
What causes departure from 'equilibrium'?

Galileo Galilei
1564 - 1642



Causality
&
Determinism

Isaac Newton
(1642-1727)



$\vec{F} = m\vec{a}$ *Effect* is proportional to the *Cause*.

Linear Response.   Principle of causality.

Equation of motion:

Rigorous mathematical relation between

position, velocity/momentum and acceleration.

Can we derive NEWTON's I law as a special

case of the II law by putting

$$\overrightarrow{F} = \vec{0} \text{ in } \overrightarrow{F} = m\vec{a}?$$

Linear Response Formalism $\boxed{\overrightarrow{F} = m\overrightarrow{a}}$

Cause-Effect Relationship

Alternative formulation of MECHANICS, based on a completely different principle:
Principle of Variation

$$\int_{t_1}^{t_2} L(q,\dot{q},t)dt = extremum$$

Alternative way to arrive at Newton's III law, following a very different approach -
- principle of translational invariance in homogeneous space.

This alternative approach illustrates an exciting path of discovering laws of nature, using invariance/symmetry principles.

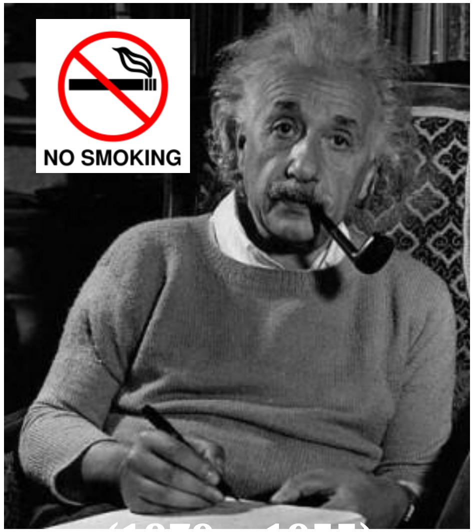
An intriguing question: Are the conservation principles consequences of the laws of nature? Or, are the laws of nature consequences of the symmetry principles that govern them?

In contemporary physics,

SYMMETRY

is placed *ahead of* LAWS OF NATURE.

This approach has its origins in the works of
Albert Einstein, Emmily Noether & Eugene Wigner.




(1882 – 1935)


(1902 – 1995)

$$\overrightarrow{F} = m\overrightarrow{a}$$

Linear Response Formalism
Cause-Effect Relationship

$$\int_{t_1}^{t_2} L(q, \dot{q}, t)\,dt = extremum$$

Principle of Variation

Lagrange's equation: *One* 2nd Order equation

Hamilton's equations: *Two* first order equations

“When we ask advice, we are usually looking for an accomplice. ”

Joseph-Louis Lagrange
(1736-1813)

William Rowan Hamilton (1805 - 1865)

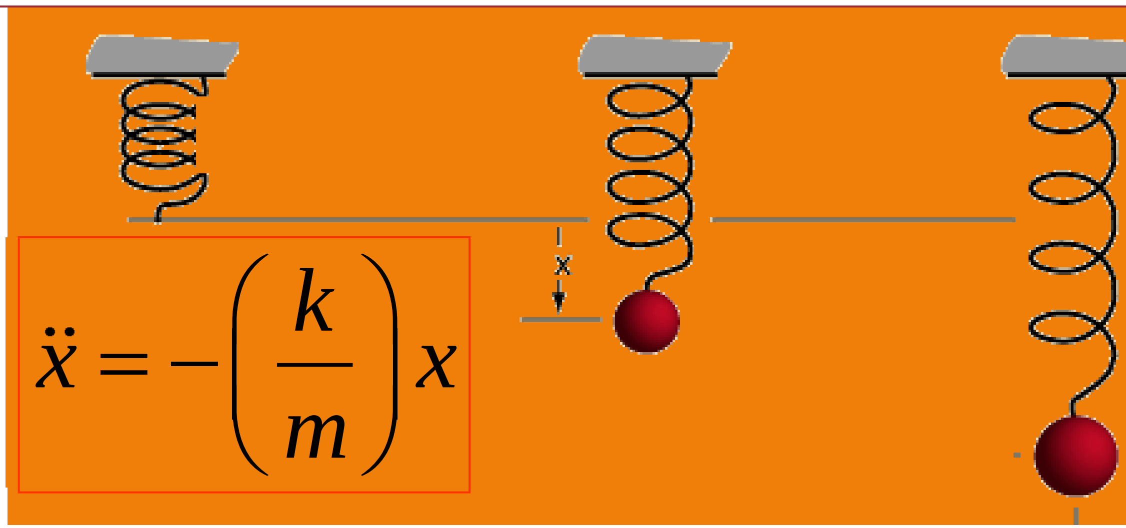
“……..we used to bring in a ‘snack’ and leave it in his study, but a brief nod of recognition of the intrusion of the chop or cutlet was often the only result, …..”

- William Edwin Hamilton (his elder son).

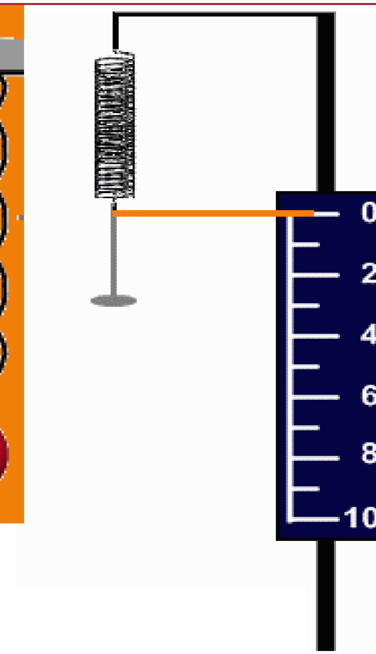
“On earth there is nothing great but man; in man there is nothing great but mind.” - Hamilton

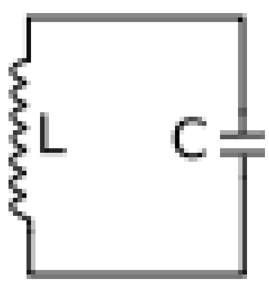
Unit 2 : Oscillations. Small oscillations. SHM.
Electromechanical analogues exhibiting SHM.
Damped harmonic oscillator, types of damping.

Driven and damped & driven harmonic oscillator.
Resonances, Quality Factor.
Waves.



$$\ddot{x} = -\left(\frac{k}{m}\right)x$$





$$\ddot{Q} = -\left(\frac{1}{LC}\right)Q$$

Resonances

Enrico Caruso
1873 - 1921

Enrico Caruso - could shatter a crystal goblet by singing a note of just the right frequency.

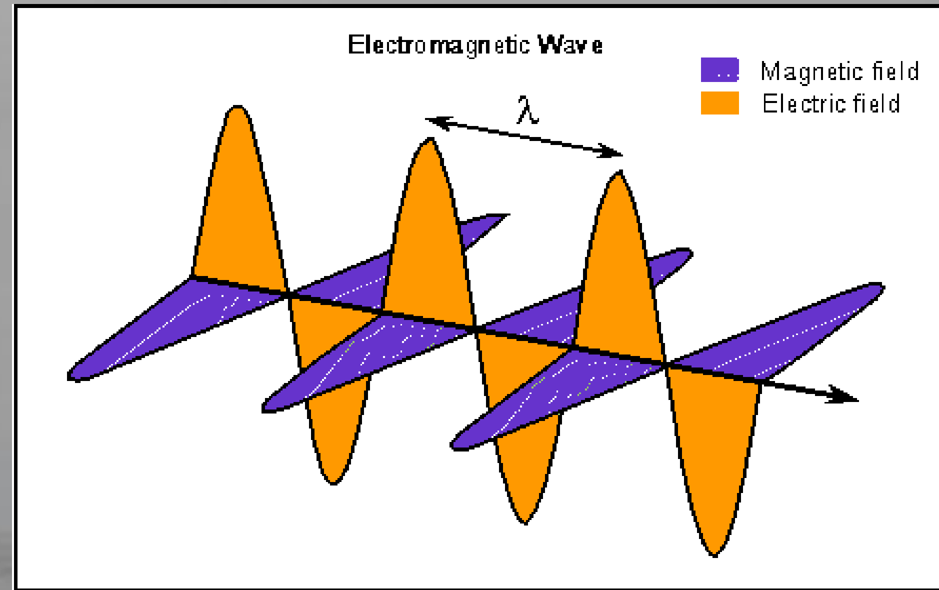
In 2005, Discovery TV Channel recruited rock singer and vocal coach Jamie Vendera to hit some crystal ware.

BAIJU / TANSEN in King AKBAR's court

He tried 12 wine glasses … and struck the lucky one that splintered at the blast of his mighty pipes.

http://www.sciam.com/article.cfm?id=fact-or-fiction-opera-singer-can-shatter-glass&sc=rss

Wave motion in one dimension.
Wave equation and travelling wave solutions.



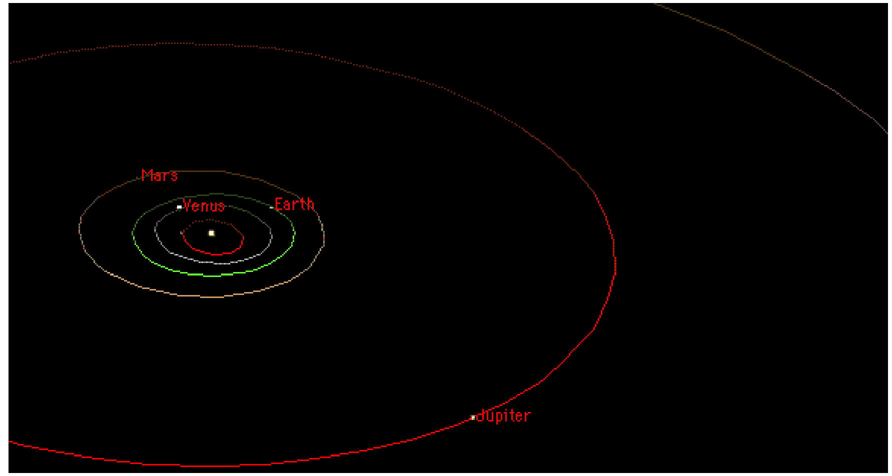
Wave velocity, group velocity and dispersion.

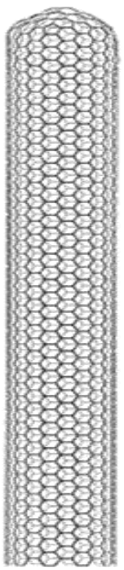
Fourier Analysis

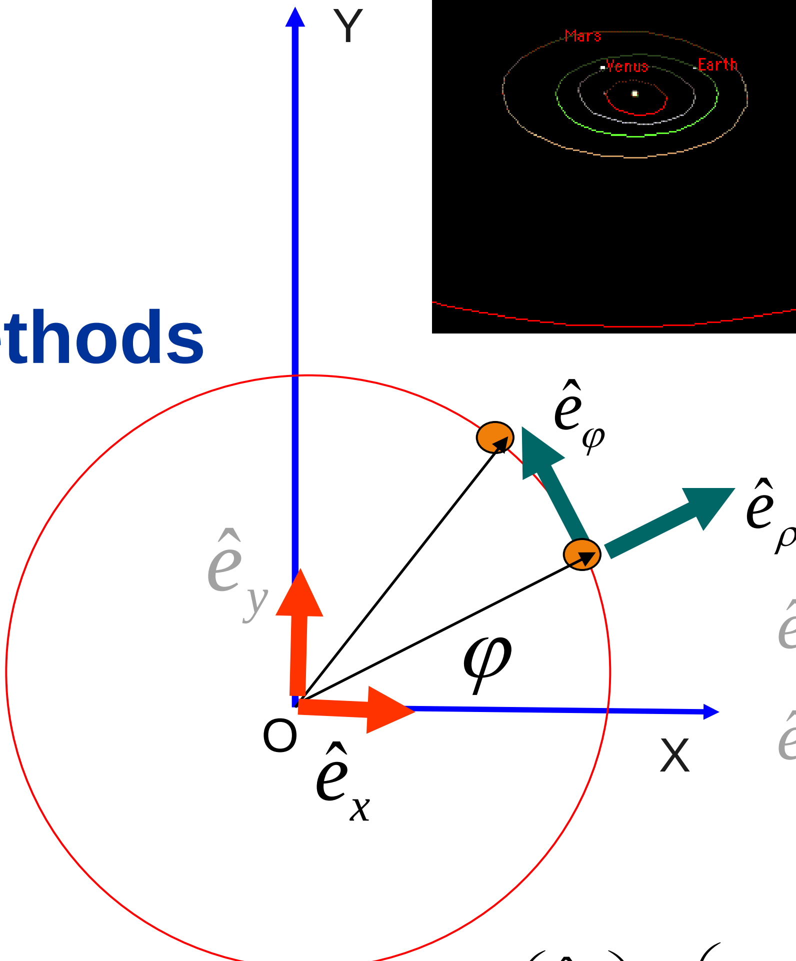
Unit 3 :

Plane polar coordinate systems.

Vector Methods







$$\hat{e}_\rho = \cos\varphi\,\hat{e}_x + \sin\varphi\,\hat{e}_y$$

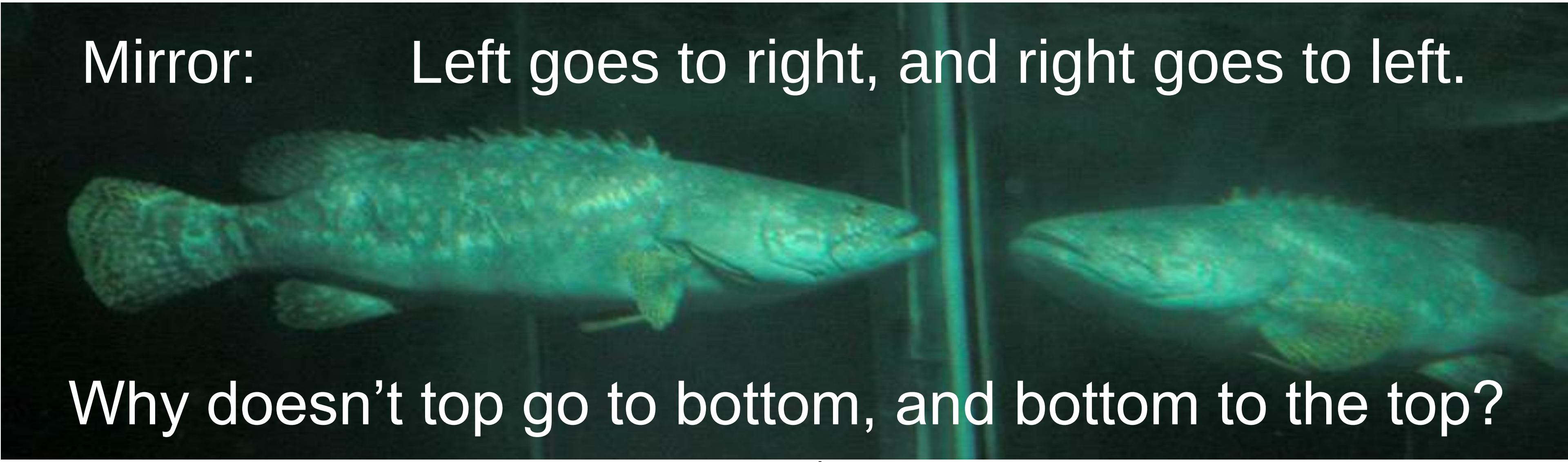
$$\hat{e}_\varphi = -\sin\varphi\,\hat{e}_x + \cos\varphi\,\hat{e}_y$$

$$\begin{pmatrix} \hat{e}_\rho \\ \hat{e}_\varphi \\ \hat{e}_z \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \hat{e}_x \\ \hat{e}_y \\ \hat{e}_z \end{pmatrix}$$

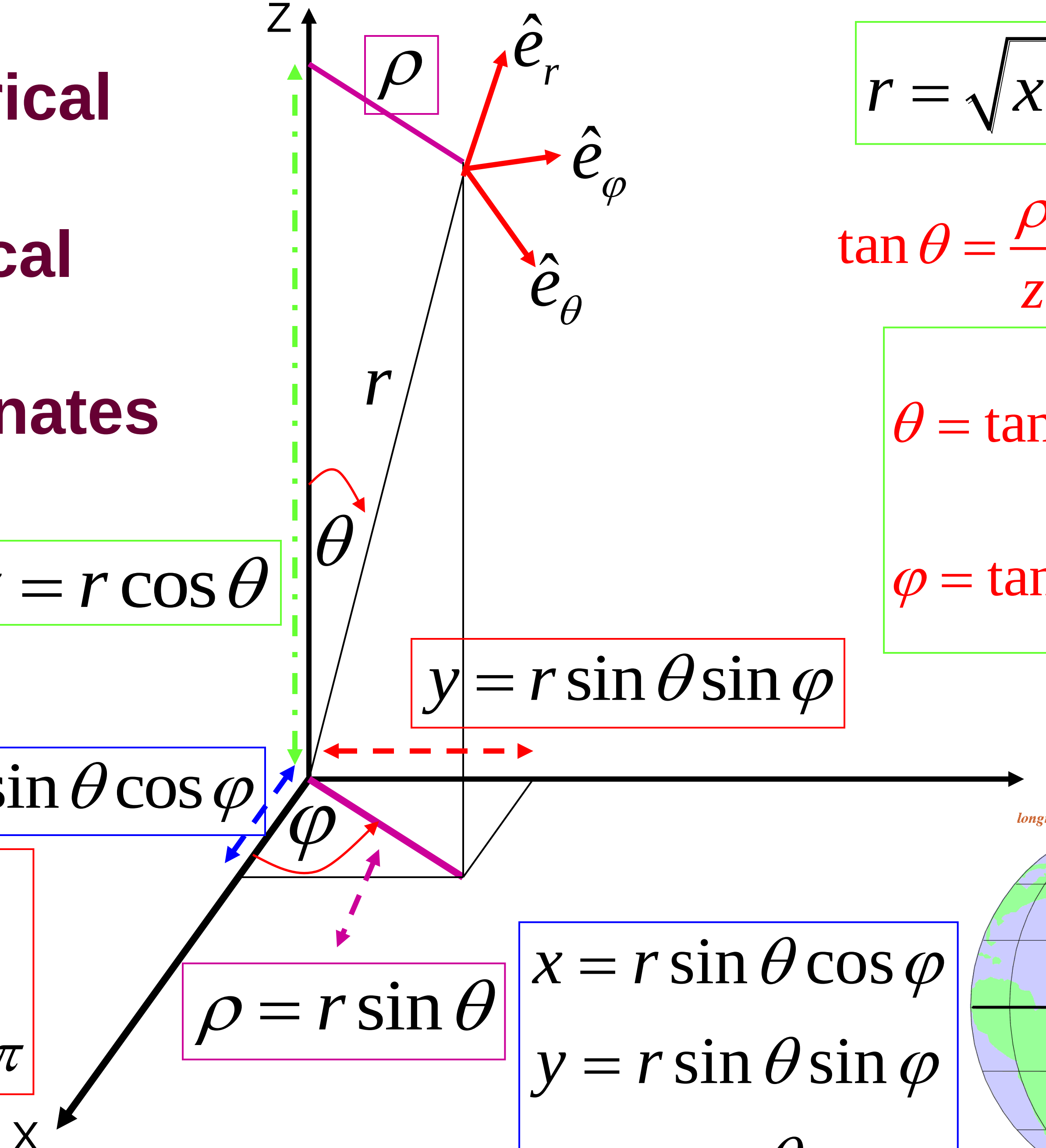
Représentation of Physical Quantitites and their Transformation
Physical Quantities that are Scalars and Vector

Mathematical Transformations of Components of a Vector

Polar and Axial (Pseudo) Vectors, Rotations and Reflections.



Cylindrical and Spherical Polar Coordinates



$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\tan\theta = \frac{\rho}{z} = \frac{\sqrt{x^2 + y^2}}{z}$$

$$\theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z}$$

$$\varphi = \tan^{-1}\frac{y}{x}$$

$$z = r\cos\theta$$

$$y = r\sin\theta\sin\varphi$$

$$x = r\sin\theta\cos\varphi$$

$$0 \le r < \infty$$

$$0 \le \theta \le \pi$$

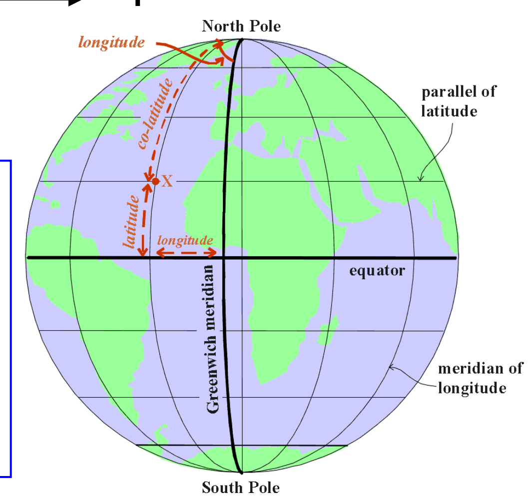
$$0 \le \varphi < 2\pi$$

$$\rho = r\sin\theta$$

$$x = r\sin\theta\cos\varphi$$

$$y = r\sin\theta\sin\varphi$$

$$z = r\cos\theta$$



Transformations of the unit vectors

$$\begin{bmatrix} \hat{e}_r \\ \hat{e}_\theta \\ \hat{e}_\varphi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \\ \cos\theta\cos\varphi & \cos\theta\sin\varphi & -\sin\theta \\ -\sin\varphi & \cos\varphi & 0 \end{bmatrix} \begin{bmatrix} \hat{e}_x \\ \hat{e}_y \\ \hat{e}_z \end{bmatrix}$$

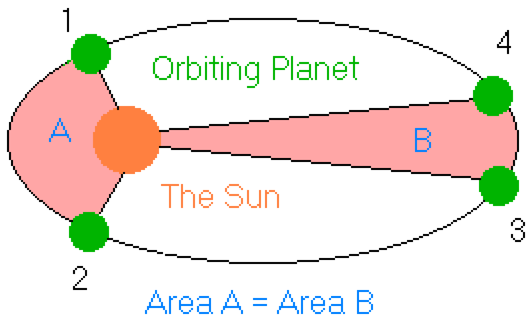
Get the inverse matrix, and write the inverse transformations.

$$\begin{bmatrix} \hat{e}_x \\ \hat{e}_y \\ \hat{e}_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\varphi & \cos\theta\cos\varphi & -\sin\varphi \\ \sin\theta\sin\varphi & \cos\theta\sin\varphi & \cos\varphi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} \hat{e}_r \\ \hat{e}_\theta \\ \hat{e}_\varphi \end{bmatrix}$$

Unit 4: Kepler Problem.

Laplace-Runge-Lenz vector, ‘Dynamical’ symmetry.
Conservation principle ↔ Symmetry relation.

Kepler Problem.
Laplace-Runge-Lenz vector, ‘Dynamical’ symmetry.
Conservation principle ↔ Symmetry relation.



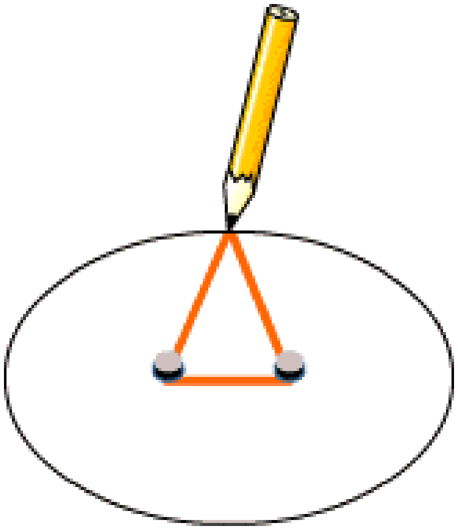
Kepler: “equal area in equal time”

Conservation of Angular Momentum : Central Force Field

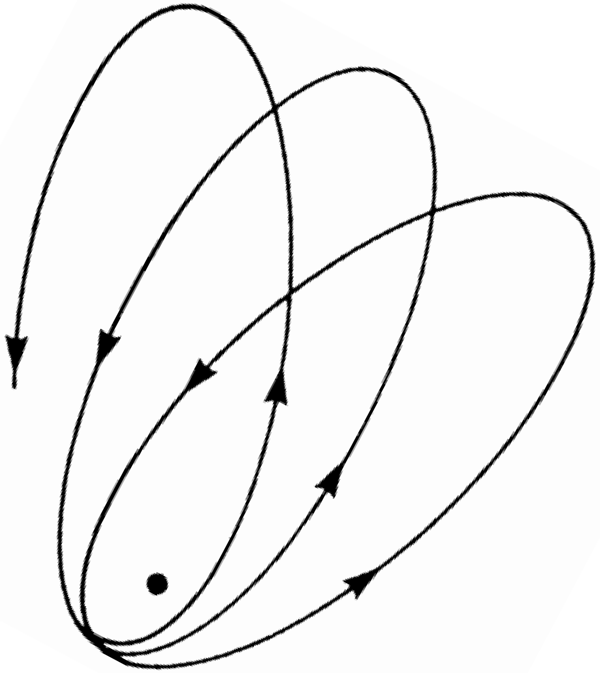
Symmetry ⟷ Conservation Law

Other than ‘energy’ and ‘angular momentum’, what *else* is conserved, *and* what is the associated symmetry?

Kepler



Angular momentum is conserved, but major-axis not fixed.

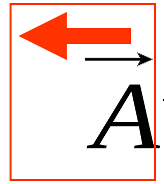


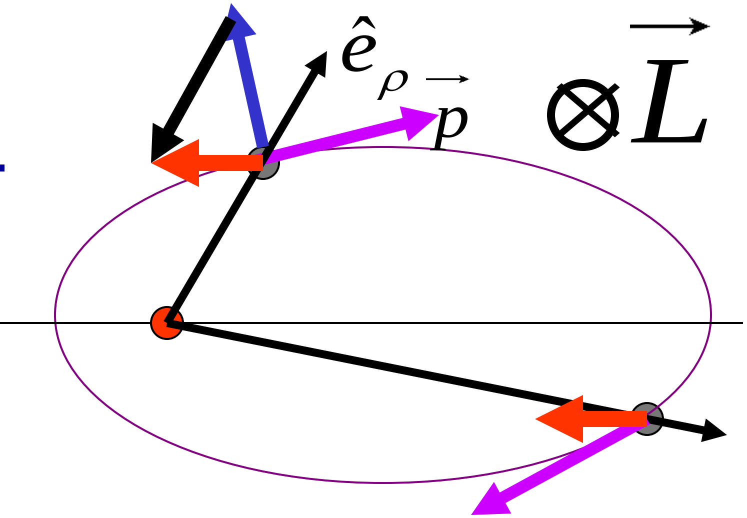
Two-body central field Kepler-Bohr problem,

Attractive force: inverse-square-law.

Laplace Runge Lenz Vector is constant for a strict 1/r potential.

$$\vec{A} = \vec{p} \times \vec{L} - mk\hat{e}_\rho$$





$$\vec{A} = mk\left[\frac{\vec{v} \times \vec{L}}{k} - \hat{e}_\rho\right] = mk\vec{a}$$

$$\vec{a} = \frac{\vec{v} \times \vec{L}}{k} - \hat{e}_\rho$$

LRL vector is often defined as $\vec{a}$ which is also, obviously, a constant.

One can show that $\left|\vec{a}\right| = e$, orbit's eccentricity.

Eccentricity vector

For $\frac{d\vec{A}}{dt} = 0$,

$$\frac{d\vec{p}}{dt} = -\frac{k}{\rho^2}\hat{e}_\rho$$

DYNAMICAL SYMMETRY

Details in Unit 4

Noether's Theorem:

'every symmetry in nature yields a conservation law and conversely, every conservation law reveals an underlying symmetry'.

Emmily Noether 1882 to 1935

SYMMETRY	CONSERVATION PRINCIPLE
Homogeneity of time	Energy
Homogeneity of space	Linear Momentum
Isotropy of Space	Angular momentum

'Dynamical Symmetry' of the inverse-square force is associated with the conservation / constancy of the eccentricity (LRL) vector.

Unit 5: Inertial and non-inertial reference frames.

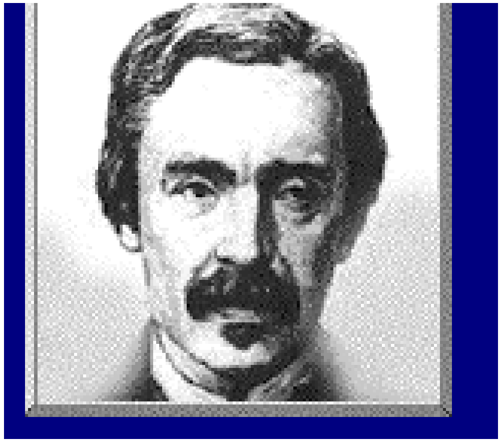
Moving coordinate systems. Pseudo forces.
Inertial and non-inertial reference frames.

Deterministic cause-effect relations in inertial frame, and their modifications in a non-inertial frame.

Real Effects of Pseudo Forces!

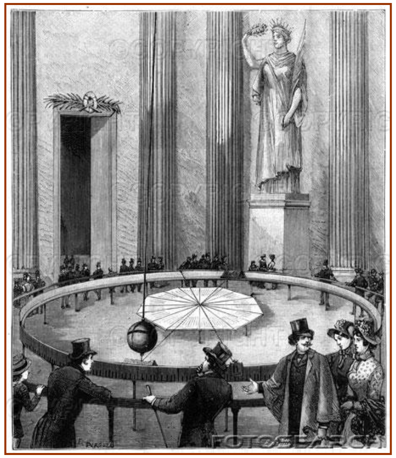
Six Flags over Georgia

Gaspard Gustave de Coriolis
1792 - 1843



“ Foo-Koh”

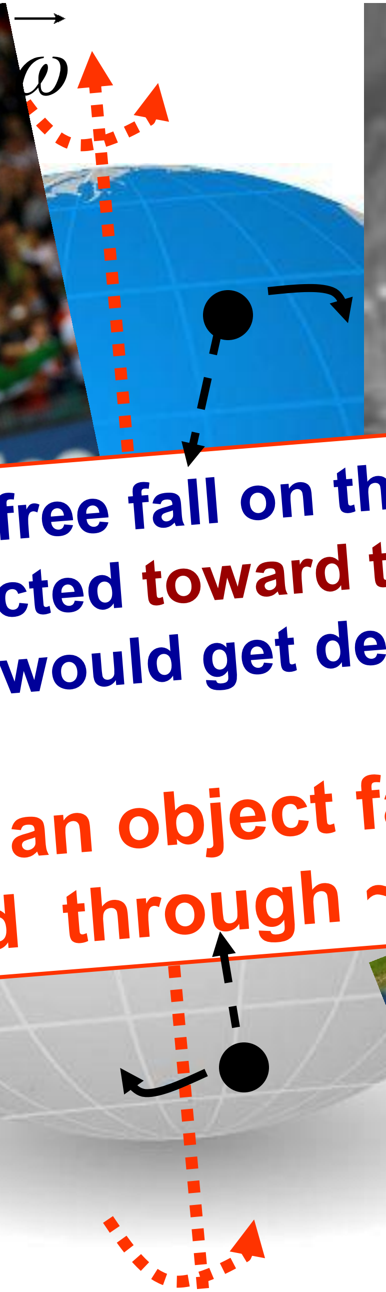
The plane of oscillation of the Foucault pendulum is seen to rotate due to the Coriolis effect. The plane rotates through one full rotation in 24 hours at poles, and in ~33.94 hours at a latitude of 45^0 (Latitude of Paris is $\sim 49^0$).

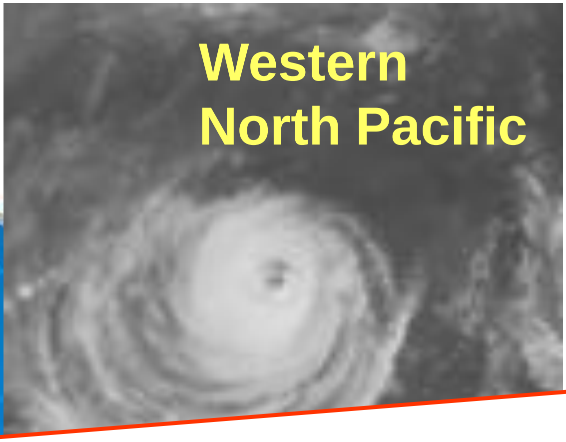


$$\vec{F}_R = \vec{F}_I - \vec{F}_{\dot{\omega}} - 2m\vec{\omega}\times\left(\frac{d}{dt}\right)_R \vec{r} - \mathrm{m}\vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right)$$

‘Leap second’ term ‘Coriolis force’ ‘Centrifugal force’

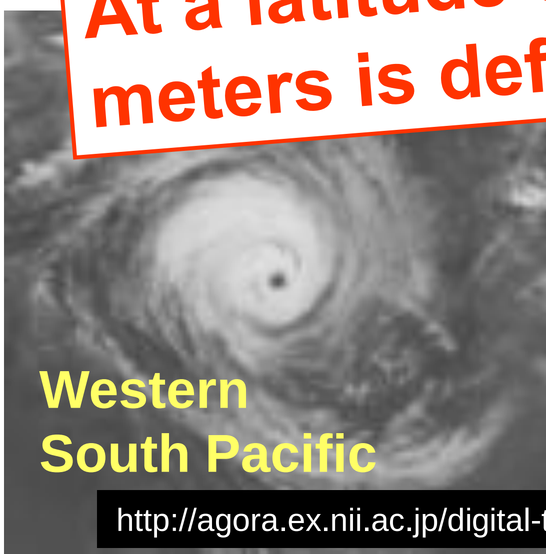
Coriolis
1792 - 1843





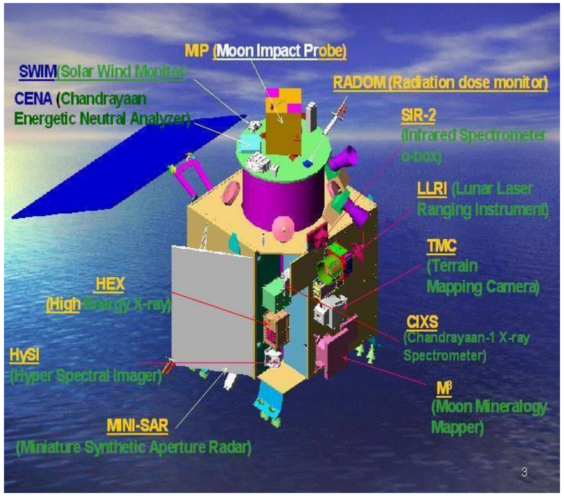
An object in a state of free fall on the Northern hemisphere gets deflected toward the East, in the Southern hemisphere would get deflected toward West!

At a latitude of 60^0 an object falling through 100 meters is deflected through ~1 cm.



http://agora.ex.nii.ac.jp/digital-typhoon/start_som_wnp.html.en

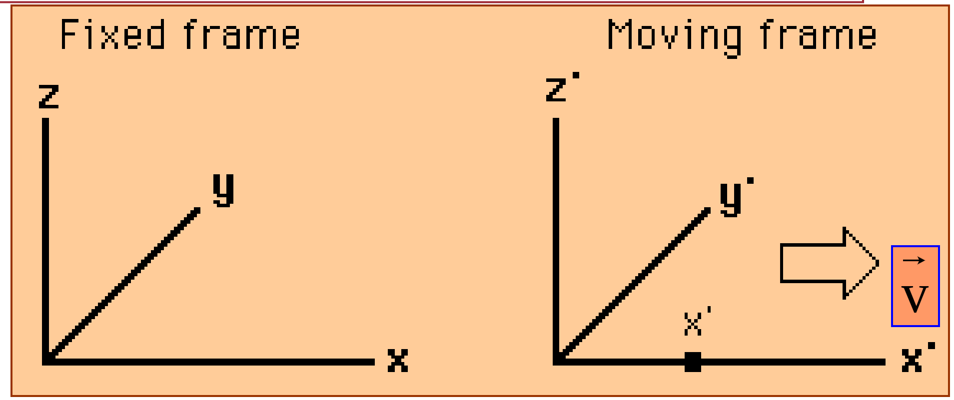
Mechanics of Flights into space.



Unit 6: Galilean & Lorentz transformations. Special Theory of Relativity.

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\gamma \to 1 \text{ as } v \to 0.$$

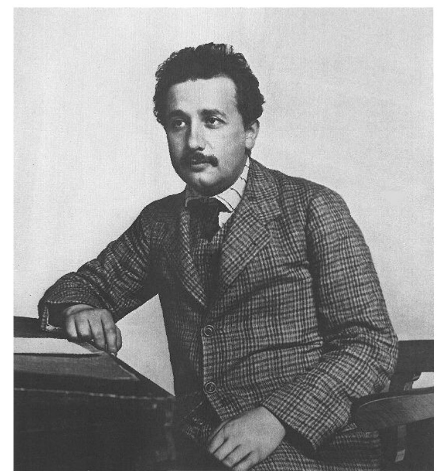


$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$

$$y' = y \qquad y = y'$$

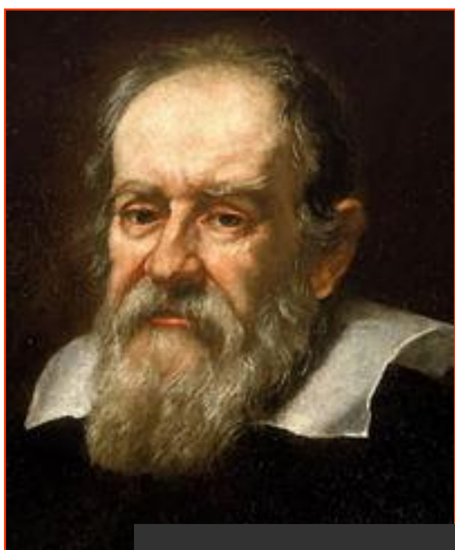
$$z' = z \qquad z = z'$$

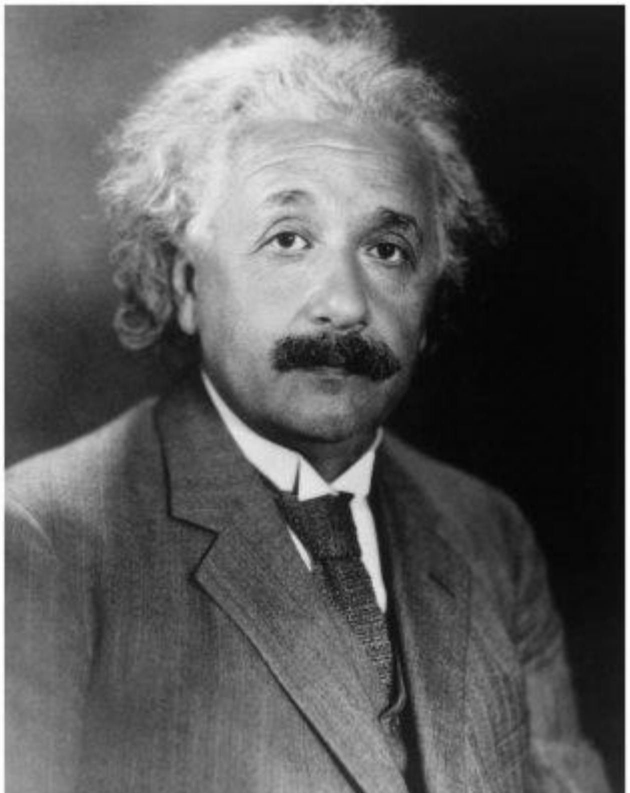
$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \qquad t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$



TWIN PARADOX !

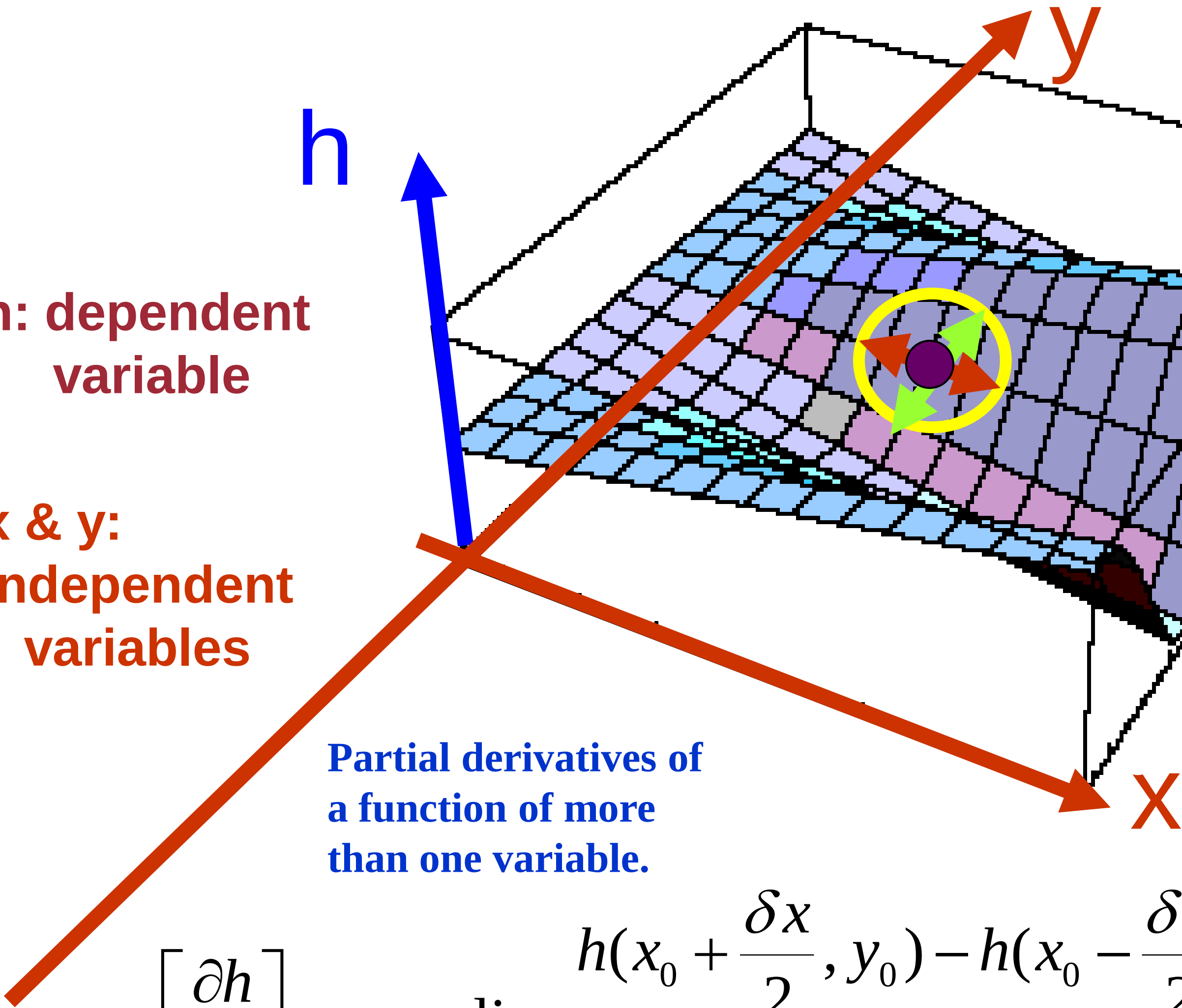
But, *just what is the paradox*?





Galileo Lorentz Einstein

Unit 7: Physical examples of fields.
Potential energy function. Potentials, Gradients, Fields



h: dependent variable

x & y: independent variables

$$\frac{d\psi}{ds} = \hat{u} \bullet \vec{\nabla}\psi$$

$$\hat{u} = \lim_{\delta s \to 0} \frac{\overrightarrow{\delta r}}{\delta s} = \frac{\overrightarrow{dr}}{ds}$$

$\delta s = \left|\overrightarrow{\delta r}\right|$, tiny increament

$ds = \left|\overrightarrow{dr}\right|$, differential increament

Partial derivatives of a function of more than one variable.

$$\left[\frac{\partial h}{\partial x}\right]_{(x_0, y_0)} = \lim_{\delta x \to 0} \frac{h(x_0 + \frac{\delta x}{2}, y_0) - h(x_0 - \frac{\delta x}{2}, y_0)}{\delta x} = \lim_{\delta x \to 0}\left[\frac{\delta h}{\delta x}\right]_{y_0}$$

Unit 8: Gauss' Law;

Equation of Continuity

Hydrodynamic

and

Electrodynamic illustrations

Johann Carl Friedrich Gauss

1777 - 1855

$$\underset{\text{volume region}}{\iiint} d\tau \left[\vec{\nabla} \bullet \vec{A}(\vec{r})\right] = \underset{\text{surface enclosing that region}}{\oiint} \vec{A}(\vec{r}) \bullet \overrightarrow{da}$$

$$\vec{\nabla} \bullet \vec{J}(\vec{r},t) + \frac{\partial \rho(\vec{r},t)}{\partial t} = 0$$

Unit 9: Physical examples of fields.
Potential energy function. Potentials, Gradients, Fields

$$\Psi = \frac{p(\vec{r})}{\rho} + \phi + \frac{|\vec{v}|^2}{2} = \text{constant for a given streamline}$$

Daniel Bernoulli
1700 - 1782

If the fluid flow is ***both*** 'steady state'

and 'irrotational', $\vec{\nabla} \times \vec{v} = \vec{\chi} = \vec{0}$

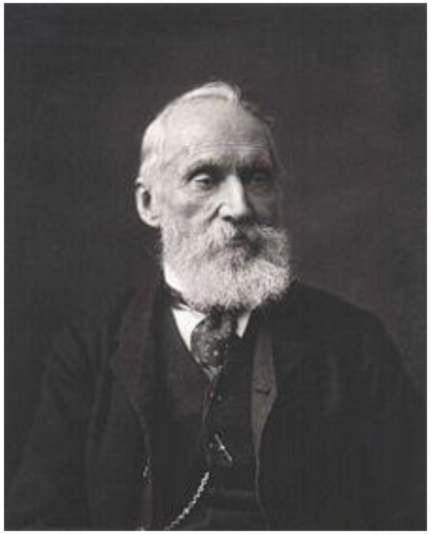
$$then \;\; \Psi = \frac{p(\vec{r})}{\rho} + \phi + \frac{|\vec{v}|^2}{2}$$

Daniel Bernoulli's Theorem

is constant for the entire velocity field in the liquid.

Curl of the vector

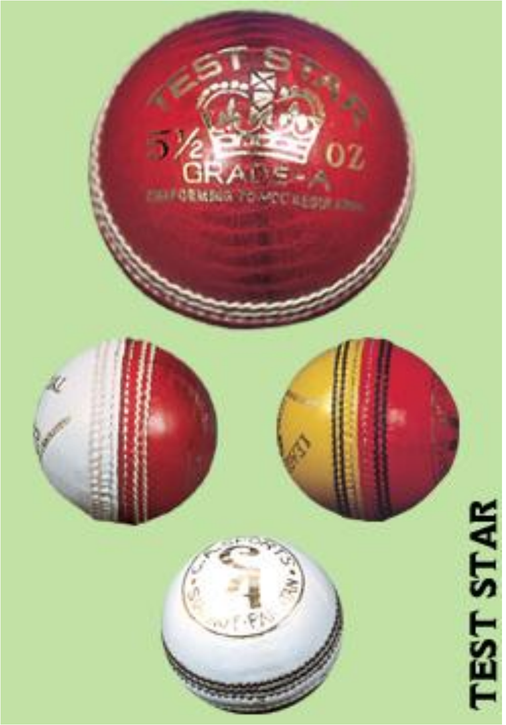
$$\oint \vec{A}(\vec{r}) \bullet d\vec{l} = \iint (\vec{\nabla} \times \vec{A}) \bullet \overrightarrow{dS}$$

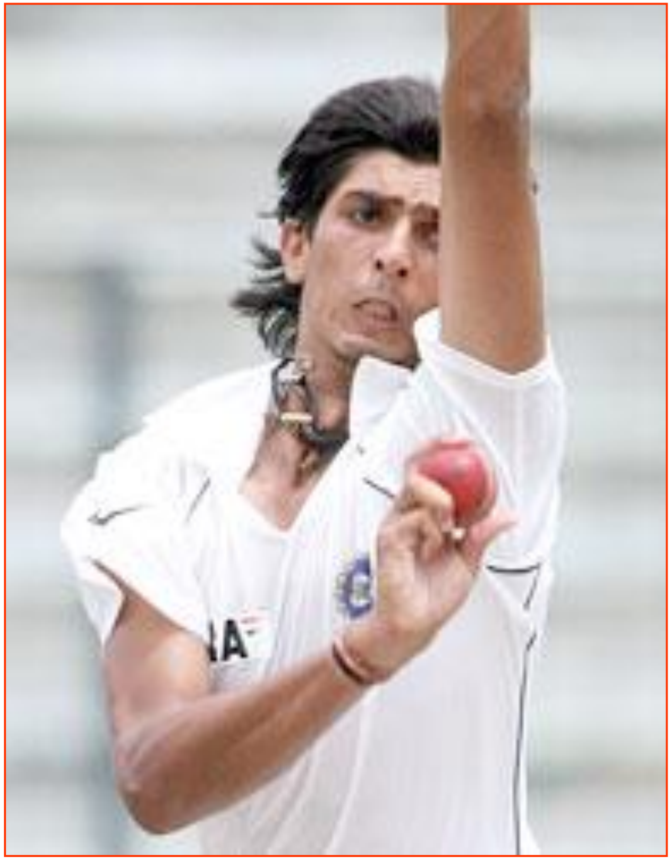


William Thomson, 1st Baron Kelvin (1824-1907)

George Gabriel Stokes (1819–1903)

This theorem is named after George Gabriel Stokes (1819–1903), although the first known statement of the theorem is by William Thomson (Lord Kelvin) and appears in a letter of his to Stokes in July 1850.



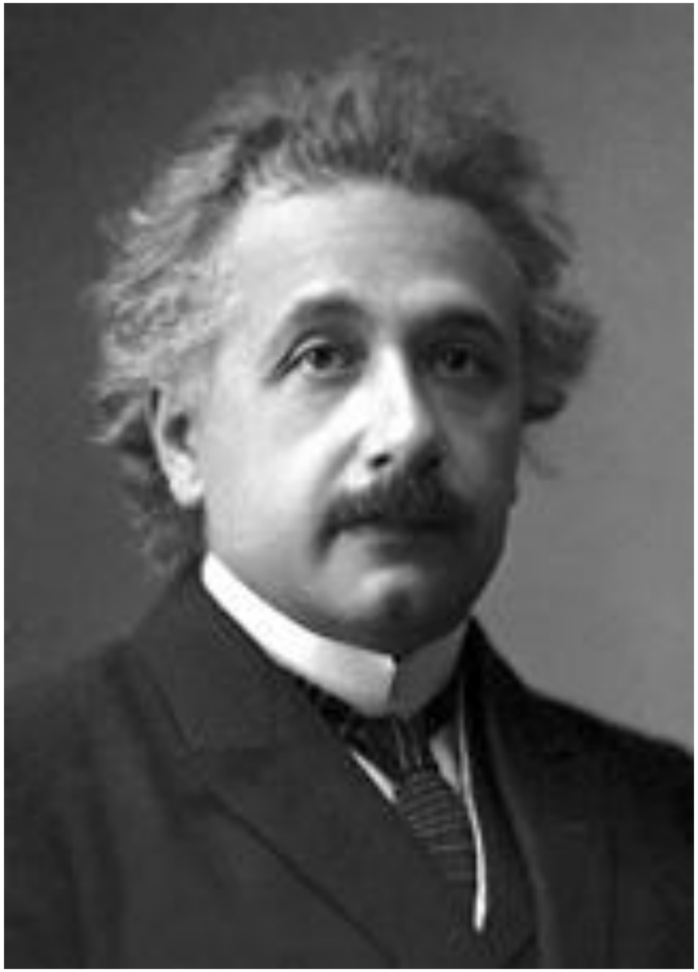


Ishant Sharma
Swing Bowling

Difference between the rough and shiny surface of a white ball is much more, and hence swings more!

Unit 10: Classical Electrodynamics and the Special Theory of Relativity.

Maxwell



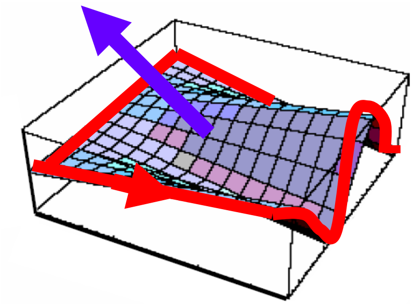
Einstein

Maxwell's equations.

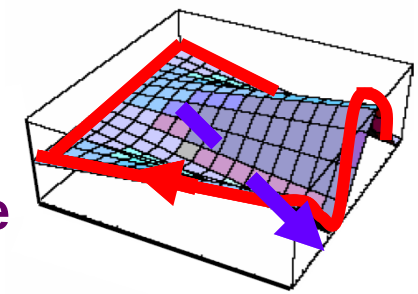
Applications in Electrodynamics

$$\iiint d\tau\left[\vec{\nabla}\bullet\vec{A}(\vec{r})\right]=\oiint\vec{A}(\vec{r})\bullet\overrightarrow{da}$$ Gauss' theorem

$$\oint\vec{A}(\vec{r})\bullet d\vec{l}=\iint(\vec{\nabla}\times\vec{A})\bullet\overrightarrow{dS}$$ Stokes' theorem



$$\oiint\vec{E}(\vec{r})\bullet\overrightarrow{dS}=\frac{Q_{enclosed}}{\varepsilon_0}$$ **Gauss' law**

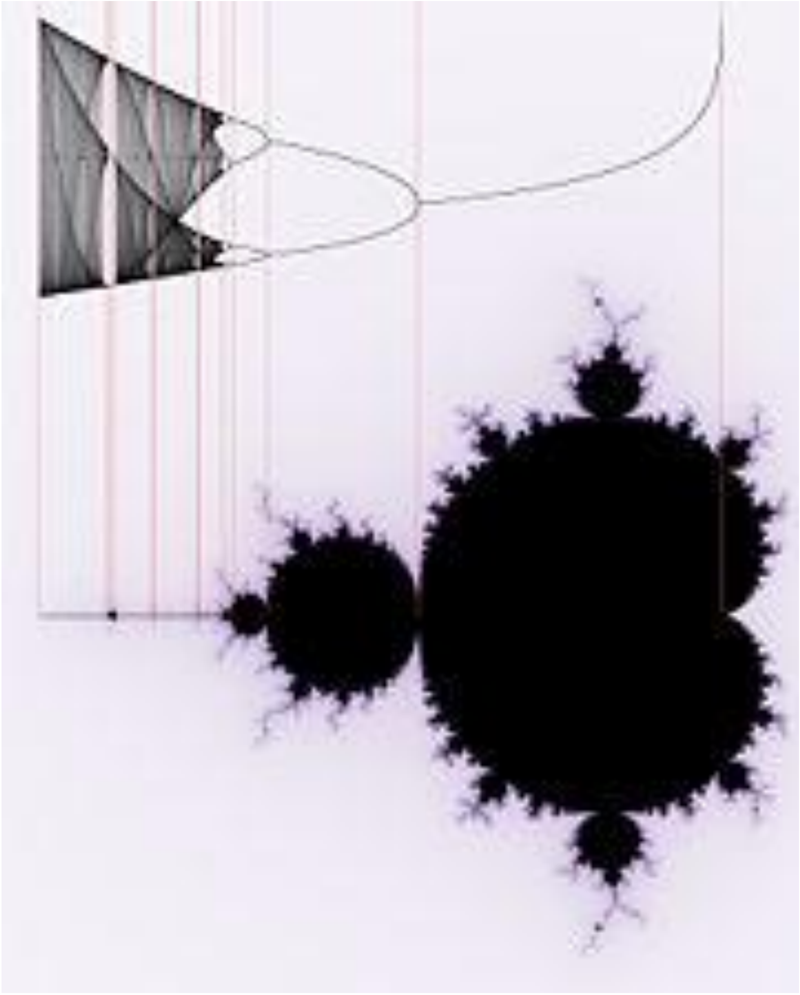


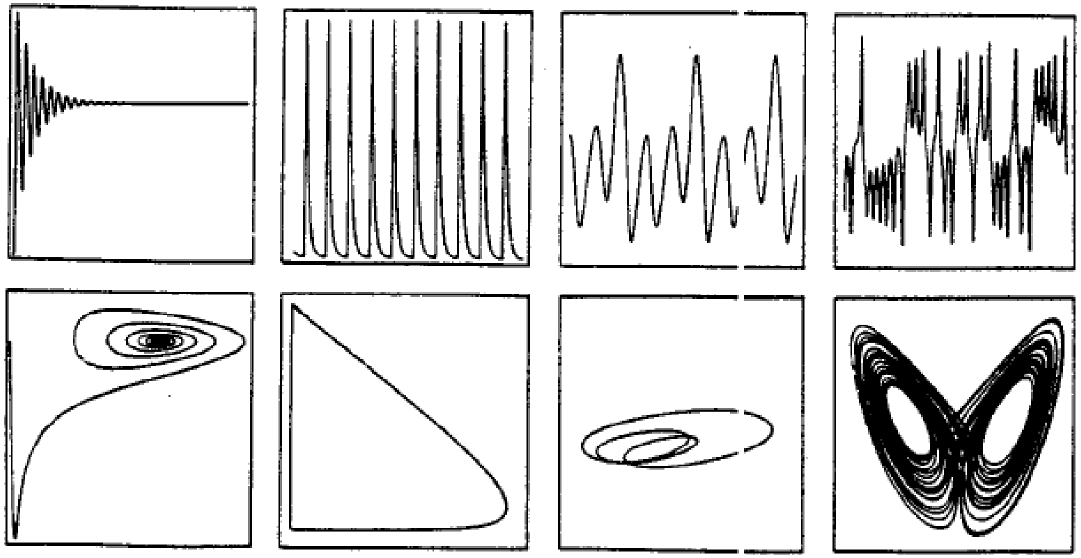
$$\oiint\vec{B}(\vec{r})\bullet\overrightarrow{dS}=0$$ **There is no Magnetic Monopole**

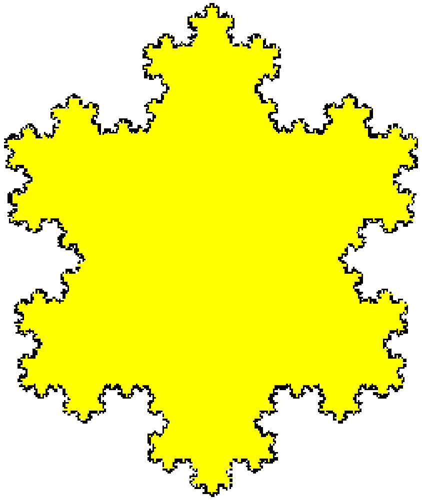
$$\oint_c\vec{E}(\vec{r})\bullet d\vec{l}=-\frac{\partial\phi_B}{\partial t}$$ **Faraday-Lenz-Maxwell**

$$\oint_c\vec{B}(\vec{r})\bullet d\vec{l}=\mu_0\varepsilon_0\frac{\partial\phi_E}{\partial t}+\mu_0 I_{enclosed}$$ **Ampere-Maxwell**

Unit 11: Logistic Map, Bifurcations, ‘Chaos’, ‘Attractor’, ‘Strange Attractor’, Fractals, ‘Self-Similarilty’ , Mandelbrot Sets…







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"One must always do what one really cannot."
- Niels Bohr (1885-1962

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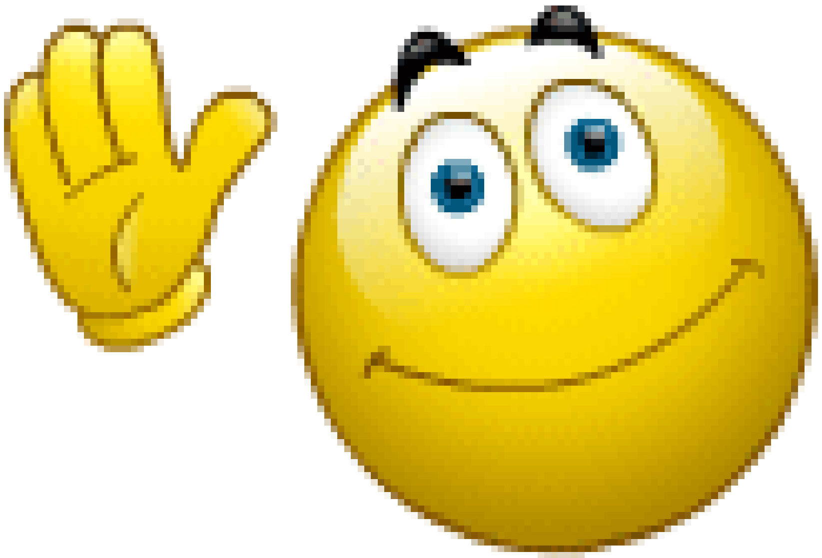

Next class: Unit 1: Equations of Motion (i)